

C Appendix: Arbitrary Initial Distribution

The solution for an arbitrary initial distribution: $N_{init}(m)$, requires solving (22) subject to different boundary conditions at $t = 0$; the terms proportional to A_m are the same, the term proportional to N_0 is replaced by the superposition of new terms describing the propagation of each bin of initial histogram:

$$\begin{aligned}
 F(m, t) &= \sum_{i=1}^{\infty} N_{init}(i) \psi_i(m, t) + A_m (\phi - \phi^{-\frac{m}{1+r}}) - \sum_{i=1}^{m-1} A_i \psi_i(m, t) \\
 \psi_i(m, t) &= \begin{cases} 0 & \text{if } m < i \\ \beta_{m-i}^i \phi^{-\frac{i}{1+r}} \left(1 - \phi^{-\frac{1}{1+r}}\right)^{m-i} & \text{for } m \geq i \end{cases} \quad (35)
 \end{aligned}$$

with the same definitions for A_m and β_n^m as before. These are derived by following by successive integration in the same way as was done in Appendix A.

The fact that $\psi_i(m, t) = 0$ for $m < i$ reflects the fact that there is no gene deletion; genes that start in bin i may either stay put or advance to bins corresponding to larger fold sizes, but will never populate bins of fold size less than i .

One important conclusion may be drawn from the full solution: all initial distributions ultimately lead to the the same limiting distribution determined by the A_m . Just as before, the dependence on the initial fold distribution $N_{init}(m)$ decays with time, leading to the same asymptotic distribution as was found for an initial distribution of N_0 folds of size 1 in (9). Of course, the details of how the crossover happens will depend on the particular form of $N_{init}(m)$.